Technical Comments

Comment on "Induced Drag and the Ideal Wake of a Lifting Wing"

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As stated in Ref. 1, the intent of that exercise was to show how the well-known results concerning induced drag in Prandtl's small-perturbation wing theory, usually derived by energy considerations, could be recovered by a careful force-impulse balance. Since the drag is second order, such first-order effects as the inclination of the plane trailing-vortex sheet have to be taken into account. Since the body of fluid is supposed to be infinitely large, Kelvin's "impulse" concept must be used, because calculations of momentum become ambiguous.

That the static pressure at the Trefftz plane far behind the wing is greater, not smaller, than the atmospheric pressure was an interesting byproduct of the study. It is seen to come from the fact that the wing and its wake, in producing lift, produce not only downward increments of velocity but also small *forward* increments.

McCutchen[2] questions these results and undertakes constructing a different theoretical model. It is difficult to comment on this effort. In the first place, the rolling up of the vortex sheet has nothing to do with this calculation and cannot have any effect on drag to second order; furthermore, some of McCutchen's concepts are foreign to fluid mechanics. He does not make his force-impulse balance clear: the total force on the system should be the net integrated pressure on the boundaries plus the external force being provided at the wing. It should be reflected in a rate of increase of the impulse of the system with consistent signs.

His Eq. (4) is wrong. The flow at the Trefftz plane is two-dimensional and lies in planes perpendicular to the vortex wake, thus inclined at an angle $2\alpha_i$ to the vertical (see Fig. 1, copied from Ref. 1). Thus a, the velocity component in the flight direction, is just $2\alpha_i w$, where w is the vertical component, and its integral is surely not the same as far upstream (viz. zero) if the wing is lifting. The error arises from careless treatment of the approximation of an infinite fluid volume; McCutchen's argument regarding the flux through the system's walls as the volume becomes infinite is a classical blunder. Thus Eq. (6) lacks its leading-order term, as do Eqs. (7) and (8). This is why Eq. (8) states such obvious nonsense—that the rate of change of momentum is the same whether momentum is being added or subtracted. McCutchen explains this by a concept of "forward flow of energy in the wake," which is surely a mystery to this writer—as is the concept of "buckling" of the wake (see Introduction).

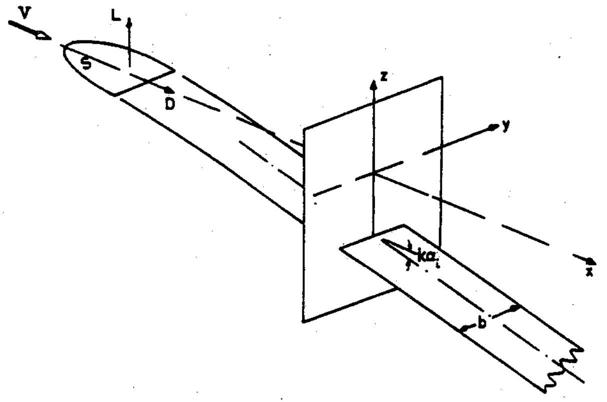

Fig. 1 Wing, trailing-vortex wake, and Trefftz plane.

I believe the little analysis of Ref. 1 is consistent and correct. I also believe that authors should have opportunities to present controversial ideas. But Prandtl's small perturbation wing theory hardly seems like a subject for controversy in 1988.

References

[1]Sears, W. R., "On Calculation of Induced Drag and Conditions Downstream of a Lifting Wing," *Journal of Aircraft*, Vol. 11, March 1974, pp. 191–192.

[2]McCutchen, C. W., "Induced Drag and the Ideal Wake of a Lifting Wing," *Journal of Aircraft*, Vol. 26, No. 5, May 1989, pp. 489–492.

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Author's Reply

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Professor Sears and I have been discussing induced drag for two years. I thank him for his continued interest.

Sears says my Eq. (4) is wrong. He writes, "The flow at the Trefftz plane is two-dimensional and lies in planes perpendicular to the vortex wake, thus inclined at an angle $2\alpha_i$ to the vertical.... Thus a, the velocity component in the flight direction, is just $2\alpha_i w$, where w is the vertical component, and its integral is surely not the same as far upstream (viz. zero) if the wing is lifting." But it is only an assumption that the forward

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